Math 602 Exam 1 (02/08/11). Max total score 50.

Your name: ____

READ THIS FIRST: Do not open the exam booklet until told to do so. Out of the first **five** problems, do any **four** (worth 10 points each). If you attempt all four problems, indicate which one is not to be graded. The exam concludes with two True/False questions worth 5 points each. You may not use the textbook or notes.

Part I: Do four out of five problems. If you attempt all five problems, indicate which one is not to be graded. Support your claims.

1. Give an example of a strictly increasing function $\alpha\colon [0,1] \to \mathbb{R}$ such that $\displaystyle\int_0^1 x\,d\alpha(x) = 5$ and $\displaystyle\int_0^1 x^2\,d\alpha(x) = 2$.

2. Define $f\colon \mathbb{R} \to \mathbb{R}$ as follows: $f(x) = (-1)^{\lfloor x^2 \rfloor}$ where $\lfloor x^2 \rfloor$ is the greatest integer that does not exceed x^2.

Prove that $\lim_{T\to\infty} \int_0^T f\,dx$ exists (as a real number).

3. Suppose that $f\colon [0,1] \to \mathbb{R}$ is integrable ($f \in \mathscr{R}$) and the set $\{x \in [0,1]\colon f(x) \neq 0\}$ is at most countable.

Prove that $\displaystyle\int_0^1 f\,dx = 0$.

4. Let $\gamma\colon [0,1] \to \mathbb{R}^2$ be a rectifiable curve such that γ attains the values $(0,0)$, $(1,0)$, and $(0,1)$, not necessarily in this order.
Prove that the length of γ is at least 2.

5. Let $\alpha\colon [0,1] \to \mathbb{R}$ be an increasing function. Suppose that $f \in \mathscr{R}(\alpha)$ on $[0,1]$. Prove that $f \in \mathscr{R}(\beta)$ on $[0,1]$, where $\beta = \alpha^3$.

Part II: True/False questions, 5 points each. You do not need to support your claims in this part.

6. "If $f\colon \mathbb{R} \to \mathbb{R}$ is differentiable at every point of $\mathbb{R}$, then for any $a < b$ we have $\int_a^b f'(x)\,dx = f(b) - f(a)$."

True ______ *False* ______

7. "If $\mathbf{f}\colon [0,2] \to \mathbb{R}^2$ is a continuous vector-valued function such that $3 \leqslant |\mathbf{f}(x)| \leqslant 5$ for every $x \in [0,2]$, then $6 \leqslant \left|\int_0^2 \mathbf{f}(x)\,dx\right| \leqslant 10$."

True ______ *False* ______