MATH 601 EXAM 3 (11/22/10). MAX TOTAL SCORE 40.

YOUR NAME: ______________________________________________

**READ THIS FIRST:** Do not open the exam booklet until told to do so. Out of the first **four** problems, do any **three** (worth 10 points each). If you attempt all four problems, indicate which one is not to be graded. The exam concludes with two True/False questions worth 5 points each. You may not use the textbook or notes. Rough work can be done on back pages of the booklet. Giving or receiving unauthorized aid during an exam is a violation of Syracuse University Academic Integrity Policy.

Part I: Do three out of four problems. If you attempt all four problems, indicate which one is not to be graded. Support your claims.

**1.** Let $X$ be a metric space. Suppose that $f\colon X \to \mathbb{R}$ is a continuous surjective function. Prove that the set $\{x \in X\colon f(x) \neq 0\}$ is not connected.

**2.** Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ is a continuous function. Prove that the set $\{f(x)\colon 0 < x < 1\}$ can be written as a countable union of closed sets.

**3.** Suppose that the power series $\sum_{n=0}^{\infty} c_n z^n$ has radius of convergence 3. Prove that the radius of convergence of the series $\sum_{n=0}^{\infty} c_{2n} z^n$ is at least 9. Also show, by an example, that this radius can be strictly greater than 9.

**4.** Suppose that $f\colon (0,1) \to \mathbb{R}$ is an increasing bounded continuous function. Prove that $f$ is uniformly continuous.

## Part II: True/False questions, 5 points each. You do not need to support your claims in this part.

**5.** "If $\sum_{n=1}^{\infty} a_n$ is a convergent series, where $a_n \in \mathbb{C}$, then $\sum_{n=1}^{\infty} \frac{a_n}{n^2}$ is also convergent."

*True* ______ *False* ______

**6.** "If $f\colon \mathbb{R} \to \mathbb{R}$ is continuous, then the function $g\colon \mathbb{R} \to \mathbb{R}$ defined by $g(x) = \frac{1}{(f(x))^2 + 1}$ is uniformly continuous."

*True* ______ *False* ______