Math 601 Exam 1 (10/25/10). Max total score 40.

Your name: ______________________________

READ THIS FIRST: Do not open the exam booklet until told to do so. Out of the first **four** problems, do any **three** (worth 10 points each). If you attempt all four problems, indicate which one is not to be graded. The exam concludes with two True/False questions worth 5 points each. You may not use the textbook or notes. Rough work can be done on back pages of the booklet. Giving or receiving unauthorized aid during an exam is a violation of Syracuse University Academic Integrity Policy.

Part I: Do three out of four problems. If you attempt all four problems, indicate which one is not to be graded. Support your claims.

1. Let X be a metric space. Suppose that $K \subset X$ is a compact set which has no limit points in X. Prove that K is a finite set.

2. Let E_n, $n = 1, 2, \ldots$, be some collection of connected subsets of a metric space X. Suppose that for each $n \in \mathbb{N}$ the intersection $E_n \cap E_{n+1}$ is nonempty. Prove that the union $\bigcup_{n=1}^{\infty} E_n$ is a connected set.

3. Let $\{p_n\}$ and $\{q_n\}$ be two sequences in a metric space X. Suppose that $\lim_{n\to\infty} d(p_n, q_n) = 0$. Prove that if $\{p_n\}$ is a Cauchy sequence, then $\{q_n\}$ is also a Cauchy sequence.

4′. Suppose that $\{x_n\}$ is a sequence of real numbers such that $\liminf_{n\to\infty} x_n^2 = 0$. Prove that $\liminf_{n\to\infty} x_n \leqslant 0$.

Part II: True/False questions, 5 points each. You do not need to support your claims in this part.

5. "If $\{x_n\}$ is a sequence of real numbers such that $\lim_{n\to\infty} |x_n - x_{n+1}| = 0$, then $\{x_n\}$ converges."

True ______ *False* ______

6. "If $\{x_n\}$ is a bounded sequence of real numbers, then

$$\limsup_{n\to\infty} |x_n| = \left|\limsup_{n\to\infty} x_n\right| \text{ ''}$$

True ______ *False* ______