MATH 601 EXAM 1 (09/27/10). MAX TOTAL SCORE 40.

YOUR NAME: _____

READ THIS FIRST: Do not open the exam booklet until told to do so. Out of the first **four** problems, do any **three** (worth 10 points each). If you attempt all four problems, indicate which one is not to be graded. The exam concludes with two True/False questions worth 5 points each. You may not use the textbook or notes. Rough work can be done on back pages of the booklet. Giving or receiving unauthorized aid during an exam is a violation of Syracuse University Academic Integrity Policy.

Part I: Do three out of four problems. If you attempt all four problems, indicate which one is not to be graded. Support your claims.

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a function such that $f(f(x)) = x + 1$ for all $x \in \mathbb{R}$. Prove that f is bijective.

2. Suppose that z and w are complex numbers such that $|z^2 - w^2| < 10$. Prove that $|z| - |w| < 4$.

3. Suppose that A is a subset $\mathbb{R}$ such that the intersection $A \cap [-n, n]$ is closed in $\mathbb{R}$ for every $n \in \mathbb{N}$. Prove that A is a closed subset of $\mathbb{R}$.

4. Let A, B, and C be subsets of some set X. Prove that

$$(A \cap B) \cup (B \cap C) \cup (C \cap A) = (A \cup B) \cap (B \cup C) \cap (C \cup A)$$

Part II: True/False questions, 5 points each. You do not need to support your claims in this part.

5. "If $\mathbf{x}$ and $\mathbf{y}$ are vectors in $\mathbb{R}^3$ such that $\mathbf{x} \cdot \mathbf{y} = 0$, then $|\mathbf{x} + \mathbf{y}| = |\mathbf{x} - \mathbf{y}|$."

True _____   *False* _____

6. "The set of all finite subsets of $\mathbb{R}$ is countable."

True _____   *False* _____