Math 601 Exam 3 (11/18/09). Max total score 40.

Your name: ______________________________

READ THIS FIRST: Do not open the exam booklet until told to do so. Out of the first **four** problems, do any **three** (worth 10 points each). If you attempt all four problems, indicate which one is not to be graded. The exam concludes with two True/False questions worth 5 points each. You may not use the textbook or notes. Rough work can be done on back pages of the booklet. Giving or receiving unauthorized aid during an exam is a violation of Syracuse University Academic Integrity Policy.

Part I: Do three out of four problems. If you attempt all four problems, indicate which one is not to be graded. Support your claims.

1. (a) Find the radius of convergence of the series $\sum_{n=0}^{\infty} 2^n z^{n^3}$.

(b) Determine the set of all complex numbers z for which the series converges absolutely.

2. Let E be a nonempty bounded subset of a metric space X with metric d. For $x \in X$ define

$$f(x) = \sup\{d(x, y) \colon y \in E\}$$

Prove that f is uniformly continuous on X.

3. Let $f\colon \mathbb{R} \to \mathbb{R}$ be a monotone function such that $f(V)$ is open for every open set $V \subset \mathbb{R}$. Prove that f is continuous on $\mathbb{R}$.

4. Given a series $\sum_{n=1}^{\infty} a_n$ with $a_n \in \mathbb{C}$, let $s_n = a_1 + \cdots + a_n$ be partial sums. We say that the series $\sum_{n=1}^{\infty} a_n$ is *super-convergent* if $\sum_{n=1}^{\infty} s_n$ is a convergent series.

(a) Prove that any super-convergent series is convergent.

(b) Give an example of a super-convergent series $\sum_{n=1}^{\infty} a_n$ such that $a_n \neq 0$ for all n.

Part II: True/False questions, 5 points each. You do not need to support your claims in this part.

5. "If the functions $f\colon (0,1) \to \mathbb{R}$ and $g\colon (0,1) \to \mathbb{R}$ are uniformly continuous, then the product fg is also uniformly continuous on $(0,1)$."

True ______ *False* ______

6. "If K is a nonempty compact connected metric space and $f\colon K \to K$ is a continuous mapping, then $f(x) = x$ for some $x \in K$."

True ______ *False* ______