MATH 601 EXAM 2 (10/16/09). MAX TOTAL SCORE 40.

YOUR NAME: _____

READ THIS FIRST: Do not open the exam booklet until told to do so. Out of the first **four** problems, do any **three** (worth 10 points each). If you attempt all four problems, indicate which one is not to be graded. The exam concludes with two True/False questions worth 5 points each. You may not use the textbook or notes. Rough work can be done on back pages of the booklet. Giving or receiving unauthorized aid during an exam is a violation of Syracuse University Academic Integrity Policy.

Part I: Do three out of four problems. If you attempt all four problems, indicate which one is not to be graded. Support your claims.

1. Let A and B be compact subsets of a metric space X. Prove that the closure of the set $A \setminus B$ is compact.

2. (a) Construct a metric d_1 on $\mathbb{R}$ such that $\mathbb{R}$ is not complete with respect to d_1.
(b) Construct a metric d_2 on $\mathbb{Q}$ such that $\mathbb{Q}$ is complete with respect to d_2.
In each case, prove the completeness or lack thereof.

3. Let X be a metric space. Suppose that there is an at most countable set $C \subset X$ such that $\overline{C} = X$. Prove that for any set $E \subset X$ there exists an at most countable set $A \subset E$ such that $E \subset \overline{A}$.

4. Prove or disprove the following statement. Any subset E of a metric space X is the intersection of some family of open subsets of X.

Part II: True/False questions, 5 points each. You do not need to support your claims in this part.

5. "If (x_n) be a sequence of real numbers such that both sequences $a_n = x_n + x_{n+1}$ and $b_n = x_n + x_{n+2}$ converge, then (x_n) converges."

True _____ *False* _____

6. "The diameter of any bounded set $E \subset \mathbb{R}$ is equal to the diameter of the interior of E."

True _____ *False* _____