Math 601 Exam 1 (09/23/09). Max total score 40.

Your name: ____

READ THIS FIRST: Do not open the exam booklet until told to do so. Out of the first **four** problems, do any **three** (worth 10 points each). If you attempt all four problems, indicate which one is not to be graded. The exam concludes with two True/False questions worth 5 points each. You may not use the textbook or notes. Rough work can be done on back pages of the booklet. Giving or receiving unauthorized aid during an exam is a violation of Syracuse University Academic Integrity Policy.

Part I: Do three out of four problems. If you attempt all four problems, indicate which one is not to be graded. Support your claims.

1. Suppose that z is a complex number such that for any $x \in \mathbb{R}$ the equality

$$|z + x| = |z - x| \tag{1}$$

holds. Prove that $\operatorname{Re} z = 0$.

Conversely, suppose that $\operatorname{Re} z = 0$. Does it follow that (1) holds for all $x \in \mathbb{R}$? Prove your assertion.

2. Let $\mathbf{x}$ and $\mathbf{y}$ be two unit vectors in $\mathbb{R}^k$, that is, $|\mathbf{x}| = 1$ and $|\mathbf{y}| = 1$. Prove that

$$|\mathbf{x} + \mathbf{y}|^2 + |\mathbf{x} - \mathbf{y}|^2 = 4$$

3. Give an example of an order on the set $\mathbb{R}^3$. Verify the order properties (trichotomy and transitivity).

4. Prove the inequality $12\,\mathrm{Re}\,z - 5\,\mathrm{Im}\,z \leqslant 13|z|$, where z is an arbitrary complex number. Give an example of z, other than 0, for which equality is attained.

Part II: True/False questions, 5 points each. You do not need to support your claims in this part.

5. "If the intervals $(a_1, b_1), \ldots, (a_9, b_9)$ on the real line are such that for any indices $1 \leqslant j < k \leqslant 9$ the intersection $(a_j, b_j) \cap (a_k, b_k)$ is not empty, then $\bigcap_{j=1}^{9} (a_j, b_j)$ is not empty."

True ______ *False* ______

6. "The set of all infinite sequences $a_1, a_2, a_3, \ldots$ such that $a_n \in \mathbb{Q}$ for all $n \in \mathbb{N}$, is countable."

True ______ *False* ______