Goal: Compress a given signal by the factor of about 10 using Fourier series. Estimate the quality of compression in terms of L^2 norm.

Data: function $f(t)$ given as samples at $t = 0, 0.02, 0.04, \ldots, 1$. Posted at http://goo.gl/r15Ow6 (link also available on Blackboard). Use the data in the row where ID matches the last two digits of your SUID number.

Method: Use the cosine Fourier series: $f(t) = \frac{1}{2}A_0 + \sum_{n=1}^{\infty} A_n \cos \pi n t$ where

$$(1) \qquad\qquad A_n = 2 \int_0^1 f(t) \cos \pi n t$$

Use the trapezoidal rule to compute the integral in (1), thus obtaining A_n for $n = 0, 1, \ldots, 15$.

The trapezoidal rule for the integral of function $g(t)$ sampled at $t = a, a+h, a+2h, \ldots, a+Nh$ is is

$$\int_a^{a+Nh} g(t)\, dt \approx h \left(\frac{g(a) + g(a + Nh)}{2} + \sum_{k=1}^{N-1} g(a + kh) \right)$$

In our situation $a = 0$, $h = 0.02$, and $N = 50$.

Out of the coefficients $A_0, \ldots, A_{15}$, identify the five largest A_n by absolute value (you may want to highlight them on the spreadsheet). Copy these values and the corresponding values of n (denoted $n_1, \ldots, n_5$ below) elsewhere on the spreadsheet: this is the compressed data.

Reconstruct the signal from the compressed data as the sum

$$A_{n_1} \cos \pi n_1 t + \cdots + A_{n_5} \cos \pi n_5 t$$

(If the term $n = 0$ is among these, do not forget $\frac{1}{2}$.)

For comparison, plot the original and reconstructed functions together.

To quantify the quality of compression, use the Parseval equality. First, calculate

$$(2) \qquad\qquad \int_0^1 f(t)^2\, dt$$

using the trapezoidal rule. Then calculate the sum on the other side of Parseval's equality, using only the coefficients you kept:

$$(3) \qquad\qquad \frac{1}{2}A_{n_1}^2 + \cdots + \frac{1}{2}A_{n_5}^2$$

For index 0, the coefficient $\frac{1}{2}$ will be replaced by $\frac{1}{4}$. The reason is that

$$\int_0^1 (A_n \cos \pi n t)^2\, dt = \frac{1}{2}A_n^2, \quad n \neq 0$$

$$\int_0^1 \left(\frac{1}{2}A_0 \right)^2 dt = \frac{1}{4}A_0^2$$

Record the ratio of (3) to (2), expressed as a percentage. This is a measure of the quality of compression.

Submit the spreadsheet on Blackboard by the end of Thursday 11/21.