Math 517 Project 3

Numerical solution of the wave equation

due 10/29

Goal: obtain a numeric approximation to the solution of the PDE $u_{tt} = c^2 u_{xx}$ with Dirichlet boundary conditions. Observe the periodicity of solution.

Data (posted on Blackboard among the grades): propagation speed c and initial velocity v (that is, $u_t(x,0) = v$, a constant).

Method: Use space step $h = 0.04$ and time step $k = 0.04$. Set up the x and t values so that they cover space interval $0 \le x \le 1$ and time interval $0 \le t \le 6$.

c	v	k	h								
??	??	0.04	0.04								
t\x		0.00	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36
0.00		initial values in this row..........									
0.04		0.00	initial + displacement due to initial velocity..........								
0.08		0.00									
0.12		0.00									
0.16		0.00			difference scheme here						
0.20		0.00									

The initial conditions are $u(x,0) = \max(0, 20x(1-3x))$ and $u_t(x,0) = v$. The first condition is enforced by filling the row for $t = 0$ with the given formula. The second is enforced by filling the second row ($t = k$) with $u(x,k) = u(x,0) + kv$.

The boundary conditions are $u(0,t) = 0$ and $u(1,t) = 0$. Enforce them by filling appropriate columns with zeros.

Use the difference scheme

$$U_j^{n+1} = 2U_j^n - U_j^{n-1} + \frac{c^2k^2}{h^2}\left(U_{j-1}^n - 2U_j^n + U_{j+1}^n\right)$$

to calculate the solution. (Here U_j^n is the approximate value of u after j space steps and n time steps from the upper left corner $x = 0, t = 0$.) Note that $k^2/h^2 = 1$ due to $k = h$.

To observe the periodicity of solution, plot its values with $x = 0.44$ and t varying from 0 to 6. (These values occupy a certain column on the spreadsheet). Determine approximate value of period T from this plot. Then plot $u(x,0)$ and $u(x,T)$ together.

Report your observations:

(a) the observed period T of the solution

(b) how T compares to the theoretical period $2/c$

(c) the degree of similarity between $u(x,0)$ and $u(x,T)$

Submit the spreadsheet on Blackboard by the end of Tuesday 10/29.