Math 517 Project 2

Numerical solution of the diffusion equation: effect of boundary conditions

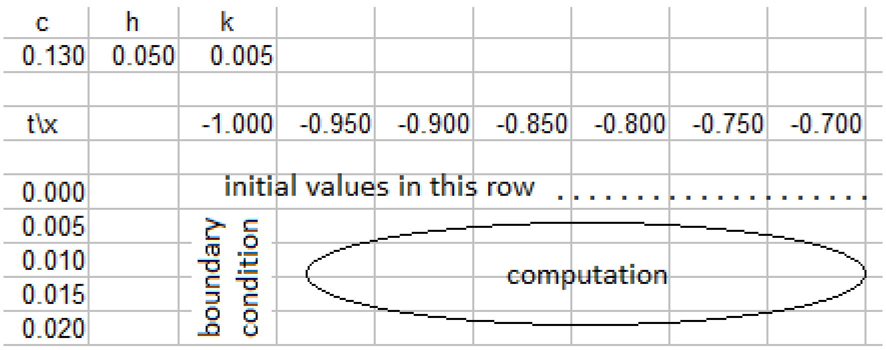
due 10/6

Goal: obtain a numeric approximation to the solution of the PDE $u_t = cu_{xx}$ with Dirichlet boundary condition at one end and Neumann boundary condition at another. Compare the results to the formula for solution on $\mathbb{R}$.

Data (posted on Blackboard among the grades): diffusion coefficient c and space step h.

Method: First, choose a value of time step k so that $2ck < h^2$ and that $1/k$ is an integer (this will make sure that $t = 1$ is achieved after some number of steps).

Then set up the x and t values, using the steps h and k, so that they cover space interval $-1 \le x \le 1$ and time interval $0 \le t \le 1$.



For the initial values of u use $u = 1/h$ when $x = 0$ and $u = 0$ otherwise.

Use the boundary conditions $u(-1,t) = 0$ and $u_x(1,t) = 0$. To enforce $u(-1,t) = 0$, fill the appropriate column with zeros. To enforce $u_x(1,t) = 0$, make the column corresponding to $x = 1$ have the same values as its neighbor to the left.

Use the difference scheme

$$U_j^{n+1} = U_j^n + \frac{ck}{h^2}\left(U_{j-1}^n - 2U_j^n + U_{j+1}^n\right)$$

to calculate the solution. (Here U_j^n is the approximate value of u after j space steps and n time steps from the upper left corner $x = -1, t = 0$. It is sometimes more accurate to think of U_j^n as the average of u over a space interval of length h.)

For comparison, evaluate the fundamental solution at $t = 1$, namely

$$\frac{1}{\sqrt{4\pi c}}\, e^{-x^2/(4c)}$$

on the same set of x-values.

Plot both the actual solution at $t = 1$ and the fundamental solution together. Compare them and state your observations about the effect of the boundary conditions on u.

Submit the spreadsheet on Blackboard by the end of Sunday 10/6.