Math 517 Project 1

Numerical solution of Burgers' equation; shock propagation speed

Goal: obtain a numeric approximation to the solution of PDE $u_t + uu_x = 0$ with discontinuous initial condition, and use it to estimate the speed of shock propagation.

Data (posted on Blackboard): velocity v, time step k, space step h.

The discrete values of x and t are labeled by $x_j = jh$ and $t_n = nk$. The approximate value of u at (x_j, t_n) is denoted by U_j^n.

Enter the initial conditions and the upwind boundary condition into your spreadsheet:

v	v	v	0	0	0	0	...
v							
v							
v							
⋮							

The top row is the initial condition, and the left side is boundary condition (which would not be needed if the spreadsheet could be infinite).

You will need about 25 columns and 50 rows.

Use conservative upwind method for Burgers' equation:

$$U_j^{n+1} = U_j^n - \frac{k}{h}\Big((U_j^n)^2/2 - (U_{j-1}^n)^2/2\Big)$$

to fill the empty cells.

Plotting the 10th, 20th, 30th, 40th and 50th row on the same chart should show a propagating shock wave (slightly diffuse due to approximation errors).

Estimate the propagation speed by comparing the position of wave on the 10th and 50th row, and taking into account the time interval between them.

Report the speed found numerically at the bottom of spreadsheet, with a remark on how it compares to the speed found theoretically.

Submit the spreadsheet on Blackboard by the end of Tuesday 9/17.