MAT 414 PROJECT 3: INVERSE LAPLACE TRANSFORM

In this project you will use two ways to find inverse Laplace transform $\mathcal{L}^{-1}$ of a given function.

- standard routine, such as `inverse_laplace` in *Sage*
- Post's inversion formula: the inverse Laplace transform of F is approximately

$$f(t) \approx \frac{(-1)^k}{k!}\left(\frac{k}{t}\right)^{k+1} F^{(k)}(k/t)$$

  where $F^{(k)}$ is the kth derivative of function F and k is large.

For this project I recommend using *Sage*, the most powerful open-source mathematics software. To use it in a browser, simply navigate to www.sagenb.org where you can either create an account or sign in with an existing account from an OpenID provider such as Google.

Let A and B be the first two nonzero digits of your SUID number. We will work with two functions:

$$G(s) = \frac{A}{(s^2+B)^3} \qquad \text{and} \qquad F(s) = \frac{A}{\sqrt{(s+B)^3+1}}$$

(put your A and B here). In my example, $A = 7$ and $B = 9$.

When working with Sage worksheet, you may want to check the box `Typeset`, which will make the output appear in more readable way. The worksheet consists of cells, each of which is evaluated by pressing Shift+Enter, or by clicking the button "evaluate".

Write down your answers to the questions underlined below, and submit them (in any format, for example MSword) on Blackboard **by the end of Tuesday, April 2**.

PART I: FUNCTION G

We will need variables t and s, so the first step is to declare them:

```
var('s t')
```

Now define function G (your numbers will be different).

```
G(s) = 7/(s^2+9)^3; G(s)
```

The part `; G(s)` is not necessary and was included just to make Sage show the formula for G. Next, find the inverse Laplace transform. The command below specifies that we transfer from variable s to variable t.

```
g(t) = inverse_laplace(G(s),s,t); g(t)
```

Again, `; g(t)` was added just to show the result.

At this point we have the inverse Laplace transform, but is it correct? Check by taking the Laplace transform of g, which should be equal to G. This transform goes from t back to s.

```
H(s) = laplace(g(t),t,s); H(s)
```

<u>Does the output look the same as the function G that you started with?</u>

Maybe it is just written in a different way. We could try to simplify H until it looks exactly as G. But it is easier to do what WebWork does to check your answers: plug the same number into both functions and compare the results. To reduce the chance of two functions agreeing by coincidence, use $s = 1.23$ instead of a "nice" number like 0 or 1. The command n produces numeric answers:

```
n(G(1.23));  n(H(1.23))
```

<u>Are these numbers the same?</u>

PART II: FUNCTION F

According to the formula for F, with my numbers I enter

```
F(s) = 7/sqrt((s+9)^3+1); F(s)
```

As before, try

```
f(t) = inverse_laplace(F(s),s,t); f(t)
```

<u>What do you get from this?</u>

Perhaps there is no reasonable formula for $\mathcal{L}^{-1}\{F\}$.

Try Post's inversion formula instead. It involves an integer k to be chosen. Begin with

```
k = 5
```

The following line implements Post's formula, with dF being the kth derivative of F.

```
dF = F.diff(k); f(t) = (-1)^k/factorial(k)*(k/t)^(k+1)*dF(k/t)
```

The output is not shown (it's an unreadably long formula), but you can see its graph on $[0, 5]$:

```
plot(f,0,5)
```

<u>Does $f(t)$ appear to have a limit as $t \to \infty$?</u>

The function f that we have is just an approximation for $\mathcal{L}^{-1}\{F(s)\}$. How accurate is it? Since f is a messy formula, there is no hope to find $\mathcal{L}\{f\}$ just by typing `laplace(f(t),t,s)`. Instead, use the definition of the Laplace transform, and as in Part I, plug $s = 1.23$.

```
numerical_integral(f(t)*exp(-1.23*t),0,+Infinity)
```

Here the integral is $\int_0^\infty f(t)\, e^{-st}\, dt$ with $s = 1.23$. *Sage* returns two numbers in parentheses: the first is the value of the integral, the second is the error of numeric integration, which we can ignore here. Compare the first number to the actual value of $F(1.23)$, namely

```
n(F(1.23))
```

<u>How close are the results?</u>

Repeat for $k = 10$, and then for $k = 15$. Repeating amounts to changing the value of k and re-evaluating the cells after that.

<u>What value of k gives the most accurate result?</u>