MAT 414 PROJECT 1: LINEAR AND NONLINEAR EQUATIONS

A first-order equation is *linear* if it can be written in the form $y' + py = q$. Here p and q can be arbitrary functions of independent variable t; they need not be linear themselves. Linearity is determined by how the unknown function y and its derivative y' appear in the equation.

In this project we compare the behavior of solutions of two similar equations, one of which is linear and the other is nonlinear.

- Linear equation: $u' = Au + t^2$ with initial condition $u(0) = 1 + B/10$.
- Nonlinear equation: $v' = At + v^2$ with initial condition $v(0) = 1 + B/10$.

Here A stands for the first digit of your SUID number, and B for the second. For example, my SUID begins with 79, so my equations are $u' = 7u + t^2$ and $v' = 7t + v^2$, with initial conditions $u(0) = 1.9$ and $v(0) = 1.9$.

The first equation could be solved by hand using the method of section 2.1, but you do not need to do it. The second equation is much harder to solve. When I asked Maple 16 to solve the second equation, it gave the following answer:

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> dsolve([diff(v(t),t)=7*t+v(t)^2,v(0)=1.9],v(t));
```

$$v(t) = \frac{7^{1/3}\left(-\dfrac{\left(19\,\pi\,3^{5/6} - 15\,7^{1/3}\,\Gamma\!\left(\frac{2}{3}\right)^2 3^{2/3}\right)\mathrm{AiryAi}\left(1, -7^{1/3}\,t\right)}{19\,\pi\,3^{1/3} + 15\,7^{1/3}\,\Gamma\!\left(\frac{2}{3}\right)^2 3^{1/6}} + \mathrm{AiryBi}\left(1, -7^{1/3}\,t\right)\right)}{-\dfrac{\left(19\,\pi\,3^{5/6} - 15\,7^{1/3}\,\Gamma\!\left(\frac{2}{3}\right)^2 3^{2/3}\right)\mathrm{AiryAi}\left(-7^{1/3}\,t\right)}{19\,\pi\,3^{1/3} + 15\,7^{1/3}\,\Gamma\!\left(\frac{2}{3}\right)^2 3^{1/6}} + \mathrm{AiryBi}\left(-7^{1/3}\,t\right)}$$

GOALS OF THE PROJECT

(1) Find approximate values of both solutions at $t = 0.05, 0.10, 0.15, \ldots, 1.95, 2$

(2) Compare the behavior of u and v.

METHOD

Starting from the initial $t = 0$, we find the equation of tangent line to the graph of u at this point, then plug $t = 0.05$ into this equation and take this value as an approximation to $u(0.05)$. Then repeat this process: find the tangent line at $t = 0.05$ and use it to find $u(0.10)$, etc. This is known as *Euler's method*. It is not very accurate but is good enough for this project.

For example: with my number, $u(0) = 1.9$. From the equation I get $u'(0) = 7u(0) + 0^2 = 13.3$. Therefore, the tangent line to the graph of u at 0 has the equation $y = 1.9 + 13.3(t - 0)$. Plugging in $t = 0.05$ I get $u(0.05) \approx 1.9 + 13.3 \cdot 0.05 \approx 2.57$. Now that $u(0.05)$ has been found, the process repeats: $u'(0.05)$, tangent line, plug in $t = 0.10$, etc.

(over)

TECHNICAL DETAILS

I recommend using spreadsheet software such as

- Microsoft Excel, available in ITS labs across campus
- OpenOffice Calc, free to download from openoffice.org,
- Google Spreadsheet (within Google Drive) which can do all computations online without you having to install anything.

To begin, download the template at goo.gl/JgHzW (or make your own copy in Google Drive).

Column A contains all necessary values of t. Columns B and C already have some values of u and u'. For example, cell B2 contains the initial value $u(0)$ (mine is 1.9, you should use yours). Cell C2 calculates $u'(0)$ by the formula

```
=7*B2+A2^2
```

according to my equation $u' = 7u + t^2$. Then cell B3 calculates an approximate value of $u(0.05)$ as

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=B2+C2*0.05
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This formula will be the same in your case, because it implements the equation of tangent line.

Then the process repeats: in C3 we have $u'(0.05)$ calculated as

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=7*B3+A3^2
```

and in B4 we have $u(0.10)$ calculated as

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=B3+C3*0.05
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These formulas can be copied to the rest of the columns B and C, either by dragging down the bottom-right corner of a cell, or just by copying and pasting.

Then work with the second equation in columns D and E.

Finally, answer the questions in Column F.

Submit your Project 1 spreadsheet on Blackboard by the end of **Wednesday, January 30**.