Table 1. Line and Surface Integrals

Integral	of a function f	of a vector field $\vec{F}$
over a curve C	$\int_C f\,ds = \int_a^b f(\vec{r}(t))\lvert\vec{r}\,'(t)\rvert\,dt$	$\int_C \vec{F}\cdot d\vec{r} = \pm\int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\,dt$
over a surface S	$\iint_S f\,dS = \iint_D f(\vec{r})\lvert\vec{r}_u\times\vec{r}_v\rvert\,dA$	$\iint_S \vec{F}\cdot d\vec{S} = \pm\iint_D \vec{F}(\vec{r})\cdot(\vec{r}_u\times\vec{r}_v)\,dA$

- $\int_C \vec{F}\cdot d\vec{r}$ can also be written as $\int_C Pdx + Qdy(+Rdz)$, where P, Q (and R) are the components of $\vec{F}$.
- The integral of a vector field over a curve has the "$-$" sign if the direction in which t increases does not match the orientation of the curve C.
- The integral of a vector field over a surface has the "$-$" sign if the normal vector $\vec{r}_u\times\vec{r}_v$ does not match the orientation of the surface S.
- Alternative ways of finding $\int_C \vec{F}\cdot d\vec{r}$:
  - if $\vec{F}$ is conservative, can use FTC: find a function f such that $\nabla f = \vec{F}$, and take the difference of its values at the endpoints of C.
  - in two dimensions: if C is a simple closed curve, can use Green's theorem

    $$\int_C Pdx + Qdy = \iint_D (Q_x - P_y)dA$$

    if C is has counterclockwise (positive) orientation; otherwise insert $-$.
- In the surface integrals, D is the parameter domain for u and v.
- If $\vec{r}(x,y) = \langle x, y, g(x,y)\rangle$, then $\vec{r}_x\times\vec{r}_y = \langle -g_x, -g_y, 1\rangle$
- Stokes' theorem relates two types of integrals of vector fields:

  $$\int_C \vec{F}\cdot d\vec{r} = \iint_S \operatorname{curl}\vec{F}\cdot d\vec{S}$$

  where C is a simple closed curve that bounds the surface S, and both are oriented according to the right-hand rule.
- The divergence theorem is an easy way to integrate a vector field over a <u>closed</u> surface:

  $$\iint_S \vec{F}\cdot d\vec{S} = \iiint_E \operatorname{div}\vec{F}\,dV$$

  where S has positive (outward) orientation, and E is the solid bounded by S.